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Modeling the impact of Obesity on Covid-19 dynamics : A stochastic and deterministic models

Maroua Amel Boubekeur[a,*], Omar Belhamiti[a]

[a] *Department of Mathematics and Computer Science, University of Mostaganem, Algeria.*

Abstract

We have witnessed that the Covid-19 epidemic in obese people, one of the epidemic diseases that is one of the important problems, has seriously affected the whole world. It is important to keep alive the concern that other epidemics may occur and to investigate the scientific approaches that may be necessary to predict the future of the epidemic.This study proposes a novel Covid-19 model that takes into account both infected individuals and specifically obese infected individuals. For the proposed model, deterministic and stochastic versions of the model have been presented, While for the stochastic model existence-uniqueness proofs . In addition, the existence of global positive solutions for the stochastic model and the conditions under which the disease becomes extinct in the population are presented. In addition to providing an insight into deterministic and stochastic approaches to the Covid-19 outbreak, these approaches have been simulated with graphical representations.

Keywords: Stochastic model; Deterministic model ; Numerical simulation.

1. Introduction

The global rise in obesity has become a substantial health concern, especially compounded by the ongoing COVID-19 pandemic. Obesity poses an elevated risk for patients, increasing the likelihood of worsened conditions in the face of viral respiratory infections. Concerns have been sparked regarding the link between obesity and the ongoing pandemic, with an examination highlighting the potential effects of obesity on respiratory physiology and the operation of immune responses. It is distinctly emphasized that individuals with obesity may be more susceptible to COVID-19, with a higher potential for transmission compared to those with lower body weight. Moreover, the presence of obesity-related comorbidities is associated with a more severe clinical course of COVID-19, heightened mortality rates, and increased hospitalization, the necessity for mechanical ventilation, and a higher likelihood of non-survival. This situation poses a

*Corresponding author
Email addresses: maroua.boubekeur.etu@univ-mosta.dz (Maroua Amel Boubekeur), omar.belhamiti@univ-mosta.dz (Omar Belhamiti)

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substantial hurdle, as individuals with excess weight, especially when accompanied by other metabolic comorbidities, are notably more susceptible to experiencing severe cases of COVID-19, contributing to a significant percentage of all hospitalizations related to the disease. Specifically, approximately 20% of hospitalized COVID-19 patients are solely affected by obesity, and when obesity is accompanied by other metabolic comorbidities like type 2 diabetes and hypertension, it contributes to nearly 60% of all COVID-19-related hospitalizations [1, 5].

Mathematical modeling is a crucial technique in the study of infectious diseases, facilitating the understanding of disease transmission mechanisms and the effective control of their spread. The literature includes numerous mathematical models examining COVID-19 infections, particularly focusing on deterministic and stochastic approaches. This study emphasizes several of these models to provide insights into the dynamic patterns of COVID-19 transmission and inform strategies for controlling the pandemic.This study presents a mathematical model of COVID-19 spread using deterministic, stochastic, and hybrid approaches. Detailed analyses and numerical simulations show the model with a rate indicator function most accurately predicts the outbreak in Russia [10]. Sha He et al. develop a discrete-time stochastic epidemic model with binomial distributions to study disease transmission. Parameters are estimated based on data from January 11 to February 13, 2020 in China. Simulations support control measure effectiveness [15]. This study by Kaustuv Chatterjee et al. examines the healthcare impact of the COVID-19 epidemic in India using a stochastic mathematical model. Results indicate that without intervention, India could face millions of cases and thousands of deaths. Immediate nonpharmacological interventions can significantly reduce the burden on healthcare resources[8], this study by Debadatta Adak et al. presents a deterministic epidemic model with four compartments to capture the progression of Covid-19. It also extends the model stochastically to consider uncertainty in disease transmission[6]. This collective body of research contributes to a deeper understanding of the dynamics and control strategies related to the COVID-19 pandemic. In this study, we adapted the deterministic model, originally formulated with classical derivatives, by introducing fractional derivatives to explore various behaviors of the model. Additionally, we proposed a stochastic model to gain a different perspective through the development of mathematical models that are effective in predicting the course of the epidemic. The deterministic models are more effective in predicting cumulative data and stochastic models are more effective in predicting daily data. The remainder of this paper is structured as follows: Section 1 delves into the formulation of the novel COVID-19 model using a deterministic approach, while Section 2 presents the stochastic model and its mathematical outcomes. Section 3 provides the numerical results along with their corresponding discussions, offering insights into the system's behavior and its implications. In Section 4, the proposed model is validated. Finally, Section 5 concludes the paper.

2. Mathematical Model

The suggested representation of the compartmental model described in our work [9] within ordinary differential equations can be expressed as follows. We present our model

$$\left\{\begin{array}{l} S'(t) = \mu - \mu S(t) - \eta S(t) - \alpha_1 B_1 (I_a(t) + I_s(t)) S(t), \\ E'(t) = \alpha_1 B_1 (I_a(t) + I_s(t)) S(t) - (\beta_1 + \beta_2 + \beta_3 + \mu) E(t), \\ I_a'(t) = \beta_1 E(t) - \mu I_a(t) - \gamma_1 I_a(t), \\ I_s'(t) = \beta_2 E(t) - (\mu + \gamma_2) I_s(t), \\ I_h'(t) = \beta_3 E(t) - (\mu + \gamma_3 + \delta_1) I_h(t), \\ O'(t) = \eta S(t) - \mu O(t) - \alpha_2 B_2 (I_a(t) + I_s(t)) O(t), \\ E_O'(t) = \alpha_2 B_2 (I_a(t) + I_s(t)) O(t) - (\beta_4 + \mu) E_O(t), \\ I_{Oh}'(t) = \beta_4 E_O(t) - (\mu + \gamma_4 + \delta_2) I_{Oh}(t), \\ R'(t) = \gamma_1 I_a(t) + \gamma_2 I_s(t) + \gamma_3 I_h(t) + \gamma_4 I_{Oh}(t) - \mu R(t), \\ D'(t) = \delta_1 I_h(t) + \delta_2 I_{Oh}(t). \end{array}\right. \tag{2.1}$$

Here, the parameter η represents the rate of new cases of obesity, while the natural birth rate of the population is denoted by μ. The transmission of the virus occurs when healthy individuals come into contact with asymptomatic and symptomatic infected individuals, with transmission rates represented by B_1 and B_2, respectively. The infectiousness probabilities of symptomatic individuals, both non-obese and obese, are denoted by α_i. The parameters β_i represent the incubation period for newly infected individuals. The recovery rates for asymptomatic, symptomatic, non-obese hospitalized, and obese hospitalized individuals are given by γ_1, γ_2, γ_3 and γ_4, respectively. The disease-induced death rates for hospitalized individuals, both non-obese and obese, are represented by δ_1 and δ_2, respectively.

3. Stochastic Model

In this section, our objective is to conduct an in-depth analysis of the COVID-19 model in obese individuals, incorporating a stochastic component. The mathematical model under scrutiny is outlined as follows:

$$\begin{cases} dS = (\mu - \mu S - \eta S - \alpha_1 B_1 (I_a + I_s) S)\, dt + \sigma_1 S(t)\, dM_1(t), \\ dE = (\alpha_1 B_1 (I_a + I_s) S - (\beta_1 + \beta_2 + \beta_3 + \mu) E)\, dt + \sigma_2 E(t)\, dM_2(t), \\ dI_a = (\beta_1 E - \mu I_a - \gamma_1 I_a)\, dt + \sigma_3 I_a(t)\, dM_3(t), \\ dI_s = (\beta_2 E - (\mu + \gamma_2) I_s)\, dt + \sigma_4 I_s(t)\, dM_4(t), \\ dI_h = (\beta_3 E - (\mu + \gamma_3 + \delta_1) I_h)\, dt + \sigma_5 I_h(t)\, dM_5(t), \\ dO = (\eta S - \mu O - \alpha_2 B_2 (I_a + I_s) O)\, dt + \sigma_6 O(t)\, dM_6(t), \\ dE_O = (\alpha_2 B_2 (I_a + I_s) O - (\beta_4 + \mu) E_O)\, dt + \sigma_7 E_o(t)\, dM_7(t), \\ dI_{Oh} = (\beta_4 E_O - (\mu + \gamma_4 + \delta_2) I_{Oh})\, dt + \sigma_8 I_{oh}(t)\, dM_8(t), \\ dR = (\gamma_1 I_a + \gamma_2 I_s + \gamma_3 I_h + \gamma_4 I_{Oh} - \mu R)\, dt + \sigma_9 R(t)\, dM_9(t), \\ dD = (\delta_1 I_h + \delta_2 I_{Oh})\, dt + \sigma_{10} D(t)\, dM_{10}(t), \end{cases} \tag{3.1}$$

where $(M_i(.))_{i=\overline{1,10}}$, are standart Brownian motions while σ_i $i = \overline{1,10}$ are stochastic constant.

3.1. Existence and Uniqueness of the Stochastic Model

In this subsection, we discuss existence and uniqueness of the following stochastic model.

Theorem 3.1. *Assuming that there are two positive constants, k_i and $\widetilde{k}_i$, that satisfy the conditions listed below:*

i) *Lipschitz condition* : $\forall\, x, \widetilde{x} \in \mathbb{R}^5, i \in \{1, .., 5\}$

$$max \left\{ |f(x,t) - f(\widetilde{x}, t)|^2,\ |g(x,t) - g(\widetilde{x}, t)|^2 \right\} \leq \widetilde{k}_i |x - \widetilde{x}|^2, \tag{3.2}$$

ii) *Linear growth condition* : $\forall\, (x,t) \in \mathbb{R}^5 \times [t_0, T]$

$$max \left\{ |f(x,t)|^2,\ |g(x,t)|^2 \right\} \leq k_i \left(1 + |x|^2\right), \tag{3.3}$$

Proof. Our goal is to demonstrate the fulfillment of Lipschitz and linear growth conditions for each function appearing on the right-hand side of the model. To accomplish this, we will begin by examining the function $F_1(t, S)$ and subsequently extend the proof to the remaining functions. We validate the initial condition for the function $F_1(t, S)$ as follows. We now verify the Lipschitz condition for $F_i(t, x_i)_{i \in \{1,2,3,4,5,6,7,8,9,10\}}$ and $G_i(t, x_i)_{i \in \{1,2,3,4,5,6,7,8,9,10\}}$

$$\begin{aligned} |F_1(t, S_1) - F_1(t, S_2)|^2 &= |-(\mu + \eta) - \alpha_1 B_1 (I_a + I_s)|^2 |S_1 - S_2|^2 \\ &\leq 2\left(2\left(\mu^2 + \eta^2\right) + 2\alpha_1^2 B_1^2 \left(|I_a|^2 + |I_s|^2\right)\right) |S_1 - S_2|^2 \\ &\leq 2\left(2\left(\mu^2 + \eta^2\right) + 2\alpha_1^2 B_1^2 \left(\sup_{0<t<T} |I_a|^2 + \sup_{0<t<T} |I_s|^2\right)\right) |S_1 - S_2|^2 \\ &\leq 4\left(\mu^2 + \eta^2 + \alpha_1^2 B_1^2 \left(\| I_a^2 \|_\infty + \| I_s^2 \|_\infty\right)\right) |S_1 - S_2|^2 \\ &\leq \widetilde{k}_1 |S_1 - S_2|^2, \end{aligned} \tag{3.4}$$

by employing the norm defined before. Thus, we have

$$|F_1(t,S_1) - F_1(t,S_2)|^2 \leq \widetilde{k_1}\,|S_1 - S_2|^2,$$

where

$$\widetilde{k_1} = 4\left(\mu^2 + \eta^2 + \alpha_1^2 B_1^2\left(\left\|I_a^2\right\|_\infty + \left\|I_s^2\right\|_\infty\right)\right).$$

We will continue our proof with the second function, $F_2(t,E)$ then, we obtain

$$\begin{aligned} |F_2(t,E_1) - F_2(t,E_2)|^2 &= |-(\beta_1+\beta_2+\beta_3+\mu)|^2\,|E_1-E_2|^2 \\ &\leq \left(\varepsilon_1 + (\beta_1+\beta_2+\beta_3+\mu)^2\right)|E_1-E_2|^2 \\ &\leq \widetilde{k_2}\,|E_1-E_2|^2, \end{aligned} \tag{3.5}$$

where

$$\widetilde{k_2} = \left(\varepsilon_1 + (\beta_1+\beta_2+\beta_3+\mu)^2\right).$$

Applying the same procedure to the other function yields,

$$\begin{aligned} |F_3(t,I_{a,1}) - F_3(t,I_{a,2})|^2 &= |-(\mu+\gamma_1)|^2\,|I_{a,1}-I_{a,2}|^2 \\ &\leq \left(\varepsilon_2 + (\mu+\gamma_1)^2\right)|I_{a,1}-I_{a,2}|^2 \\ &\leq \widetilde{k_3}\,|I_{a,1}-I_{a,2}|^2, \end{aligned} \tag{3.6}$$

$$\begin{aligned} |F_4(t,I_{s,1}) - F_4(t,I_{s,2})|^2 &= |-(\mu+\gamma_2)|^2\,|I_{s,1}-I_{s,2}|^2 \\ &\leq \left(\varepsilon_3 + (\mu+\gamma_2)^2\right)|I_{s,1}-I_{s,2}|^2 \\ &\leq \widetilde{k_4}\,|I_{s,1}-I_{s,2}|^2, \end{aligned} \tag{3.7}$$

$$\begin{aligned} |F_5(t,I_{h,1}) - F_5(t,I_{h,2})|^2 &= |(\mu+\gamma_3+\delta_1)|^2\,|I_{h,1}-I_{h,2}|^2 \\ &\leq \left(\varepsilon_4 + (\mu+\gamma_3+\delta_1)^2\right)|I_{h,1}-I_{h,2}|^2 \\ &\leq \widetilde{k_5}\,|I_{h,1}-I_{h,2}|^2, \end{aligned} \tag{3.8}$$

$$\begin{aligned} |F_6(t,O_1) - F_6(t,O_2)|^2 &= |-(\mu+\alpha_2 B_2(I_a+I_s))|^2\,|O_1-O_2|^2 \\ &\leq 2\left(\mu^2 + 2\alpha_2^2 B_2^2\left(I_a^2+I_s^2\right)\right)|O_1-O_2|^2 \\ &\leq 2\left(\mu^2 + 2\alpha_2^2 B_2^2\left(\sup_{0<t<T}|I_a|^2 + \sup_{0<t<T}|I_s|^2\right)\right)|O_1-O_2|^2 \\ &\leq 2\left(\mu^2 + 2\alpha_2^2 B_2^2\left(\left\|I_a^2\right\|_\infty + \left\|I_s^2\right\|_\infty\right)\right)|O_1-O_2|^2 \\ &\leq \widetilde{k_6}\,|O_1-O_2|^2, \end{aligned} \tag{3.9}$$

$$\begin{aligned} |F_7(t,E_{O,1}) - F_7(t,E_{O,2})|^2 &= |-(\beta_4+\mu)|^2\,|E_{O,1}-E_{O,2}|^2 \\ &\leq \left(\varepsilon_5 + (\beta_4+\mu)^2\right)|E_{O,1}-E_{O,2}|^2 \\ &\leq \widetilde{k_7}\,|E_{O,1}-E_{O,2}|^2, \end{aligned} \tag{3.10}$$

$$\begin{aligned} |F_8(t,I_{Oh,1}) - F_8(t,I_{Oh,2})|^2 &= |-(\mu+\gamma_4+\delta_2)|^2\,|I_{Oh,1}-I_{Oh,2}|^2 \\ &\leq \left(\varepsilon_6 + (\mu+\gamma_4+\delta_2)^2\right)|I_{Oh,1}-I_{Oh,2}|^2 \\ &\leq \widetilde{k_8}\,|I_{Oh,1}-I_{Oh,2}|^2, \end{aligned} \tag{3.11}$$

$$\begin{aligned} |F_9(t,R_1) - F_9(t,R_2)|^2 &= |-\mu^2| \, |R_1 - R_2|^2 \\ &\leq (\mu^2 + \varepsilon_7) |R_1 - R_2|^2 \\ &\leq \widetilde{k_9} |R_1 - R_2|^2, \end{aligned} \tag{3.12}$$

$$|F_{10}(t,D_1) - F_{10}(t,D_2)|^2 \leq \widetilde{k_{10}} |D_1 - D_2|^2. \tag{3.13}$$

For the fuction $G(x,t)$, we have

$$|G(x,t) - G(\widetilde{x},t)|^2 \leq \sigma_i |x - \widetilde{x}|^2.$$

and the Lipschitz condition is satisfied

$$\begin{aligned} |F_i(t,x_i^1) - F_i(t,x_i^2)|^2 &< \widetilde{k}_i \left(|x_i^1 - x_i^2|^2\right), \forall t \in [0,T], \\ |G_i(t,x_i^1) - G_i(t,x_i^2)|^2 &< \widetilde{k}_i \left(|x_i^1 - x_i^2|^2\right), \forall t \in [0,T]. \end{aligned} \tag{3.14}$$

To advance in the proof, we aim to establish the satisfaction of the second condition outlined in the theorem. To achieve this objective, we will follow a comparable procedure as previously described, commencing with the function $F_1(t,S)$.

For $\forall t \in [0,T]$, we get

$$\begin{aligned} |F_1(t,S)|^2 &= |\mu - (\mu+\eta)S - \alpha_1 B_1 (I_a + I_s) S|^2 \\ &\leq 2\left(|\mu|^2 + |(\mu+\eta)S + \alpha_1 B_1 (I_a+I_s) S|^2\right) \\ &\leq 2\left(|\mu|^2 + 4(\mu^2+\eta^2)|S|^2 + 4\alpha_1^2 B_1^2 \left(|I_a|^2 + |I_s|^2\right)|S|^2\right) \\ &\leq 2\left(|\mu|^2 + 4(\mu^2+\eta^2)|S|^2 + 4\alpha_1^2 B_1^2 \left(\sup_{0<t<T}|I_a|^2 + \sup_{0<t<T}|I_s|^2\right)|S|^2\right) \\ &\leq 2\left(|\mu|^2 + 4(\mu^2+\eta^2)|S|^2 + 4\alpha_1^2 B_1^2 \left(\|I_a\|^2 + \|I_s\|^2\right)|S|^2\right) \\ &\leq 2|\mu|^2 \left(1 + \frac{4(\mu^2+\eta^2) + 4\alpha_1^2 B_1^2 \left(\|I_a\|^2 + \|I_s\|^2\right)}{|\mu|^2}|S|^2\right), \end{aligned} \tag{3.15}$$

where $k_1 = 2|\mu|^2, p_1 = \dfrac{4(\mu^2+\eta^2)^2|S|^2 + 4\alpha_1^2 B_1^2 \left(\|I_a\|^2 + \|I_s\|^2\right)|S|^2}{|\mu|^2} < 1.$

Then we have

$$|F_1(t,S)|^2 \leq k_1 \left(1 + |S|^2\right). \tag{3.16}$$

$$\begin{aligned} |F_2(t,E)|^2 &= |\alpha_1 B_1 (I_a+I_s) S - (\beta_1+\beta_2+\beta_3+\mu) E|^2 \\ &\leq 2\left(2\alpha_1^2 B_1^2 \left(|I_a|^2 + |I_s|^2\right)|S|^2 + (\beta_1+\beta_2+\beta_3+\mu)^2 |E|^2\right) \\ &\leq 2\left(2\alpha_1^2 B_1^2 \left(\sup_{0<t<T}|I_a|^2 + \sup_{0<t<T}|I_s|^2\right)\sup_{0<t<T}|S|^2 + (\beta_1+\beta_2+\beta_3+\mu)^2 |E|^2\right) \\ &\leq 2\left(2\alpha_1^2 B_1^2 \left(\|I_a\|_\infty^2 + \|I_s\|_\infty^2\right)\|S\|_\infty^2 + (\beta_1+\beta_2+\beta_3+\mu)^2 |E|^2\right) \\ &\leq 4\alpha_1^2 B_1^2 \left(\|I_a\|_\infty^2 + \|I_s\|_\infty^2\right)\|S\|_\infty^2 \left(1 + \frac{(\beta_1+\beta_2+\beta_3+\mu)^2}{2\alpha_1^2 B_1^2 \left(\|I_a\|_\infty^2 + \|I_s\|_\infty^2\right)\|S\|_\infty^2}|E|^2\right) \\ &\leq k_2\left(1 + |E|^2\right), \end{aligned} \tag{3.17}$$

noting that $k_2 = 4\alpha_1^2 B_1^2 \left(\|I_a\|_\infty^2 + \|I_s\|_\infty^2 \right) \|S\|_\infty^2$ and $p_2 = \dfrac{(\beta_1 + \beta_2 + \beta_3 + \mu)^2}{2\alpha_1^2 B_1^2 \left(\|I_a\|_\infty^2 + \|I_s\|_\infty^2 \right) \|S\|_\infty^2} < 1$. Then

$$\begin{aligned} |F_3(t, I_a)|^2 &= |\beta_1 E - (\mu + \gamma_1) I_a|^2 \\ &\leq 2\left(\beta_1^2 |E|^2 + (\mu + \gamma_1)^2 |I_a|^2 \right) \\ &\leq 2\left(\beta_1^2 \sup_{0<t<T} |E|^2 + (\mu + \gamma_1)^2 |I_a|^2 \right) \\ &\leq 2\left(\beta_1^2 \|E\|^2 + (\mu + \gamma_1)^2 |I_a|^2 \right) \\ &\leq 2\beta_1^2 \|E\|^2 \left(1 + \frac{(\mu + \gamma_1)^2}{\beta_1^2 \|E\|^2} |I_a|^2 \right) \\ &\leq k_3 \left(1 + |I_a|^2 \right), \end{aligned} \tag{3.18}$$

where $k_3 = 2\beta_1^2 \|E\|^2$ and $p_3 = \dfrac{(\mu + \gamma_1)^2}{\beta_1^2 \|E\|^2} < 1$.

$$\begin{aligned} |F_4(t, I_s)|^2 &= |\beta_2 E - (\mu + \gamma_2) I_s|^2 \\ &\leq 2\left(\beta_2^2 |E|^2 + (\mu + \gamma_2)^2 |I_s|^2 \right) \\ &\leq 2\left(\beta_2^2 \sup_{0<t<T} |E|^2 + (\mu + \gamma_2)^2 |I_s|^2 \right) \\ &\leq 2\left(\beta_2^2 \|E\|^2 + (\mu + \gamma_2)^2 |I_s|^2 \right) \\ &\leq 2\beta_2^2 \|E\|^2 \left(1 + \frac{(\mu + \gamma_2)^2}{\beta_2^2 \|E\|^2} |I_s|^2 \right) \\ &\leq k_4 \left(1 + |I_s|^2 \right), \end{aligned} \tag{3.19}$$

where $k_4 = 2\beta_2^2 \|E\|^2$ and $p_4 = \dfrac{(\mu + \gamma_2)^2}{\beta_2^2 \|E\|^2} < 1$.

Then

$$\begin{aligned} |F_5(t, I_h)|^2 &= |\beta_3 E - (\mu + \gamma_3 + \delta_1) I_h|^2 \\ &\leq 2\left(\beta_3^2 |E|^2 + (\mu + \gamma_3 + \delta_1)^2 |I_h|^2 \right) \\ &\leq 2\left(\beta_3^2 \sup_{0<t<T} |E|^2 + (\mu + \gamma_3 + \delta_1)^2 |I_h|^2 \right) \\ &\leq 2\left(\beta_3^2 \|E\|_\infty^2 + (\mu + \gamma_3 + \delta_1)^2 |I_h|^2 \right) \\ &\leq 2\beta_3^2 \|E\|_\infty^2 \left(1 + \frac{(\mu + \gamma_3 + \delta_1)^2}{\beta_3^2 \|E\|_\infty^2} |I_h|^2 \right) \\ &\leq k_5 \left(1 + |I_h|^2 \right), \end{aligned} \tag{3.20}$$

where $k_5 = 2\beta_3^2 \|E\|_\infty^2$ and $p_5 = \frac{(\mu+\gamma_3+\delta_1)^2}{\beta_3^2\|E\|_\infty^2} < 1$.

$$\begin{aligned} |F_6(t,O)|^2 &= |\eta S - (\mu O + \alpha_2 B_2 (I_a + I_s) O)|^2 \\ &\le 3\left(\eta^2 |S|^2 + \mu^2 |O|^2 + 2\left(\alpha_2^2 B_2^2 \left(I_a^2 + I_s^2\right)\right) |O|^2\right) \\ &\le 3\left(\eta^2 \sup_{0<t<T} \left|S^2\right| + \mu^2 |O|^2 + 2\alpha_2^2 B_2^2 \left(\sup_{0<t<T} |I_a|^2 + \sup_{0<t<T} |I_s|^2\right) |O|^2\right) \\ &\le 3\left(\eta^2 \|S\|_\infty^2 + \left(\mu^2 + 2\alpha_2^2 B_2^2 \left(\left\|I_a^2\right\|_\infty + \left\|I_s^2\right\|_\infty\right)\right) |O|^2\right) \\ &\le 3\eta^2 \left\|S^2\right\|_\infty \left(1 + \frac{\left(\mu^2 + 2\alpha_2^2 B_2^2 \left(\left\|I_a^2\right\|_\infty + \left\|I_s^2\right\|_\infty\right)\right)}{\eta^2 \left\|S^2\right\|_\infty} |O|^2\right) \\ &\le k_6 \left(1 + |O|^2\right), \end{aligned} \tag{3.21}$$

where $k_6 = 3\eta^2 \left\|S^2\right\|_\infty$and $p_6 = \dfrac{\left(\mu^2 + 2\alpha_2^2 B_2^2 \left(\left\|I_a^2\right\|_\infty + \left\|I_s^2\right\|_\infty\right)\right)}{\eta^2 \left\|S^2\right\|_\infty} < 1$.

$$\begin{aligned} |F_7(t,E_O)|^2 &= |\alpha_2 B_2 (I_a + I_s) O - (\beta_4 + \mu) E_O|^2 \\ &\le 2\left(2\alpha_2^2 B_2^2 \left(\left|I_a^2\right| + \left|I_s^2\right|\right) \left|O^2\right| + (\beta_4 + \mu)^2 |E_O|^2\right) \\ &\le 2\left(2\alpha_2^2 B_2^2 \left(\sup_{0<t<T} \left|I_a^2\right| + \sup_{0<t<T} \left|I_s^2\right|\right) \sup_{0<t<T} \left|O^2\right| + (\beta_4 + \mu)^2 |E_O|^2\right) \\ &\le 2\left(2\alpha_2^2 B_2^2 \left(\left\|I_a^2\right\|_\infty + \left\|I_s^2\right\|_\infty\right) \left\|O^2\right\|_\infty + (\beta_4 + \mu)^2 |E_O|^2\right) \\ &\le 4\alpha_2^2 B_2^2 \left(\left\|I_a^2\right\|_\infty + \left\|I_s^2\right\|_\infty\right) \left\|O^2\right\|_\infty \left(1 + \frac{(\beta_4 + \mu)^2}{2\alpha_2^2 B_2^2 \left(\|I_a^2\|_\infty + \|I_s^2\|_\infty\right) \|O^2\|_\infty} |E_O|^2\right) \\ &\le k_7 \left(1 + |E_O|^2\right), \end{aligned} \tag{3.22}$$

where $k_7 = 4\alpha_2^2 B_2^2 \left(\left\|I_a^2\right\|_\infty + \left\|I_s^2\right\|_\infty\right) \left\|O^2\right\|_\infty$ and $p_7 = \dfrac{(\beta_4 + \mu)^2}{2\alpha_2^2 B_2^2 \left(\|I_a^2\|_\infty + \|I_s^2\|_\infty\right) \|O^2\|_\infty} < 1$.

$$\begin{aligned} |F_8(t,I_{Oh})|^2 &= |\beta_4 E_O - (\mu + \gamma_4 + \delta_2) I_{Oh}|^2 \\ &\le 2\left(\beta_4 \left|E_O^2\right| + |\mu + \gamma_4 + \delta_2|^2 |I_{Oh}|^2\right) \\ &\le 2\left(\beta_4 \sup_{0<t<T} \left|E_O^2\right| + |\mu + \gamma_4 + \delta_2|^2 |I_{Oh}|^2\right) \\ &\le 2\left(\beta_4 \left\|E_O^2\right\| + |\mu + \gamma_4 + \delta_2|^2 |I_{Oh}|^2\right) \\ &\le 2\beta_4 \left\|E_O^2\right\|_\infty \left(1 + \frac{(\mu + \gamma_4 + \delta_2)^2}{\beta_4 \left\|E_O^2\right\|_\infty} |I_{Oh}|^2\right) \\ &\le k_8 \left(1 + |I_{Oh}|^2\right), \end{aligned} \tag{3.23}$$

where $k_8 = 2\beta_4 \left\| E_O^2 \right\|_\infty$ and $p_8 = \dfrac{(\mu+\gamma_4+\delta_2)^2}{\beta_4 \left\| E_O^2 \right\|_\infty} < 1$.

$$\begin{aligned}
|F_9(t,R)|^2 &= |\gamma_1 I_a + \gamma_2 I_s + \gamma_3 I_h + \gamma_4 I_{Oh} - \mu R|^2 \\
&\leq 5\left(\gamma_1^2 \left|I_a^2\right| + \gamma_2^2 I_s^2 + \gamma_3^2 \left|I_h^2\right| + \gamma_4^2 \left|I_{Oh}^2\right| + \mu^2 |R|^2\right) \\
&\leq 5\left(\gamma_1^2 \left\|I_a^2\right\| + \gamma_2^2 \left\|I_s^2\right\| + \gamma_3^2 \left\|I_h^2\right\| + \gamma_4^2 \left\|I_{Oh}^2\right\| + \mu^2 |R|^2\right) \\
&\leq 5\left(\gamma_1^2 \left\|I_a^2\right\|_\infty + \gamma_2^2 \left\|I_s^2\right\|_\infty + \gamma_3^2 \left\|I_h^2\right\|_\infty + \gamma_4^2 \left\|I_{Oh}^2\right\|_\infty\right) \\
&\quad \times \left.1 + \frac{\mu^2}{\gamma_1^2 \|I_a^2\|_\infty + \gamma_2^2 \|I_s^2\|_\infty + \gamma_3^2 \left\|I_h^2\right\|_\infty + \gamma_4^2 \left\|I_{Oh}^2\right\|_\infty} |R|^2\right) \\
&\leq k_9\left(1 + |R|^2\right),
\end{aligned} \tag{3.24}$$

where

$$\begin{cases}
k_9 = 5\left(\gamma_1^2 \left\|I_a^2\right\|_\infty + \gamma_2^2 \left\|I_s^2\right\|_\infty + \gamma_3^2 \left\|I_h^2\right\|_\infty + \gamma_4^2 \left\|I_{Oh}^2\right\|_\infty\right) \\
p_9 = \dfrac{\mu^2}{\gamma_1^2 \|I_a^2\|_\infty + \gamma_2^2 \|I_s^2\|_\infty + \gamma_3^2 \left\|I_h^2\right\|_\infty + \gamma_4^2 \left\|I_{Oh}^2\right\|_\infty} < 1.
\end{cases}$$

$$\begin{aligned}
|F_{10}(t,D)|^2 &= |\delta_1 I_h + \delta_2 I_{Oh}|^2 \\
&\leq 2\left(\delta_1^2 |I_h|^2 + \delta_2^2 |I_{Oh}|^2\right) \\
&\leq 2\left(\delta_1^2 \left\|I_h^2\right\|_\infty + \delta_2^2 \left\|I_{Oh}^2\right\|_\infty\right) \\
&\leq 2\left(\delta_1^2 \left\|I_h^2\right\| + \delta_2^2 \left\|I_{Oh}^2\right\|\right)\left(1 + |D|^2\right) \\
&\leq k_{10}\left(1 + |D|^2\right).
\end{aligned} \tag{3.25}$$

where $k_{10} = 2\left(\delta_1^2 \left\|I_h^2\right\| + \delta_2^2 \left\|I_{Oh}^2\right\|\right)$

For the function $G(t,x)$, we obtain

$$\begin{aligned}
|G(t,S)|^2 &= \sigma_1^2 |S|^2 \leq \sigma_1^2\left(1 + |S|^2\right) \leq k_1\left(1 + |S|^2\right) \\
|G(t,E)|^2 &\leq \sigma_2^2\left(1 + |E|^2\right) \leq k_2\left(1 + |E|^2\right), \\
|G(t,I_a)|^2 &\leq \sigma_3^2\left(1 + |I_a|^2\right) \leq k_3\left(1 + |I_a|^2\right), \\
|G(t,I_s)|^2 &\leq \sigma_4^2\left(1 + |I_s|^2\right) \leq k_4\left(1 + |I_s|^2\right), \\
|G(t,I_h)|^2 &\leq \sigma_5^2\left(1 + |I_h|^2\right) \leq k_5\left(1 + |I_h|^2\right), \\
|G(t,O)|^2 &\leq \sigma_6^2\left(1 + |O|^2\right) \leq k_6\left(1 + |O|^2\right), \\
|G(t,E_o)|^2 &\leq \sigma_7^2\left(1 + |E_o|^2\right) \leq k_7\left(1 + |E_o|^2\right), \\
|G(t,I_{oh})|^2 &\leq \sigma_8^2\left(1 + |I_{oh}|^2\right) \leq k_8\left(1 + |I_{oh}|^2\right), \\
|G(t,R)|^2 &\leq \sigma_9^2\left(1 + |R|^2\right) \leq k_9\left(1 + |R|^2\right), \\
|G(t,D)|^2 &\leq \sigma_{10}^2\left(1 + |D|^2\right). \leq k_{10}\left(1 + |D|^2\right),
\end{aligned} \tag{3.26}$$

where $k_i = \sigma_i^2$.

The system has a unique solution under the condition

$$\max\{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\} < 1.$$

□

In the following subsection, we will examine the overall positivity of the stochastic mode. To accomplish this objective, we will utilize the theorem outlined below [14, 4, 12, 3, 16, 2].

3.2. Global positive solution of stochastic model

Theorem 3.2. *Let us consider a stochastic Covid-19 model with initial data*

$$\widetilde{T}(0) = (S(0), E(0), I_a(0), I_s(0), I_h(0), O(0), E_O(0), I_{Oh}(0), R(0), D(0)) \in \mathbb{R}_+^{10}. \tag{3.27}$$

Then, there exists a non-negative solution

$$\widetilde{T}(t) = (S(t), E(t), I_a(t), I_s(t), I_h(t), O(t), E_O(t), I_{Oh}(t), R(t), D(t)) \tag{3.28}$$

for $\forall\ t \geq 0$ *such that the solution of the system will remain within* $\mathbb{R}_+^{10}$ *almost surely.*

Proof. Since the coefficients of the system satisfy the local Lipschitz condition, then the stochastic system has a unique solution $\widetilde{T}(t) \in \mathbb{R}_+^{10}$ for $t \in [0, \tau_e]$, where τ_e is known as the explosion time[14, 4, 12, 3, 16, 2]]. Our aim is to prove that this solution is global. In other words, we need to demonstrate that $\tau_e = \infty$ it probability one for each $\gamma \geq \gamma_0$, $\gamma \in \mathbb{N}$, we define the stopping

$$\tau_\gamma = \left\{ t \in (0, \tau_e) : \min\left\{\widetilde{T}(t)\right\} \leq \frac{1}{\gamma} \text{ or } \max\left\{\widetilde{T}(t)\right\} \geq \gamma \right\}, \tag{3.29}$$

assuming that γ is sufficiently large such that $\widetilde{T}(0) \in \left\{\frac{1}{\tau_\gamma}, \tau_\gamma\right\}$.By the definition of the stopping time, τ_γ is increasing, where $\gamma \to \infty$. Let $\tau_\infty = \lim_{\gamma\to\infty} \tau_\gamma$ then $\tau_\infty \leq \tau_e$. Then we must show that $\tau_\infty = \infty$ with probability one, then $\tau_e = \infty$ almost surely and $\widetilde{T}(t) \in \mathbb{R}_+^{10}$ for each $t \geq 0$. Where it is incorrect, there exists a pair $\psi(t) \geq 0$ and $\varepsilon \in (0, 1)$, such that

$$P\{r_\infty \leq \psi\} > \varepsilon. \tag{3.30}$$

By the Lyapunov functional methodology, one can conclude that the stochastic system admits the existence of a global positive solution. Thus, we define a C^2-function $\Phi : \mathbb{R}_+^{10} \to \mathbb{R}$ given by

$$\begin{aligned} \Phi(S, E, I_a, I_s, I_h, O, E_O, I_{Oh}, R, D) &= S + E + I_a + I_s + I_h + O + E_O + I_{Oh} + R + D \\ &\quad -10 - \begin{pmatrix} \ln S + \ln E + \ln I_a + \ln I_s + \ln I_h \\ \ln O + \ln E_O + \ln I_{Oh} + \ln R + \ln D \end{pmatrix}. \end{aligned} \tag{3.31}$$

Obviously, $\Phi \geq 0$ since $c - 1 - \ln c \geq 0$ for all $c > 0$. Applying the Ito formula, we can obtain the following result

$$
\begin{aligned}
& d\Phi\left(S, E, I_a, I_s, I_h, O, E_O, I_{Oh}, R, D\right) \qquad (3.32)\\
= & \left(1-\frac{1}{S}\right) dt+\sigma_1(S-1)\, dM_1(t) \\
& +\left(1-\frac{1}{E}\right) dt+\sigma_2(E-1)\, dM_2(t) \\
& +\left(1-\frac{1}{I_a}\right) dt+\sigma_3\left(I_a-1\right) dM_3(t) \\
& +\left(1-\frac{1}{I_s}\right) dt+\sigma_4\left(I_s-1\right) dM_4(t) \\
& +\left(1-\frac{1}{I_h}\right) dt+\sigma_5\left(I_h-1\right) dM_5(t) \\
& +\left(1-\frac{1}{O}\right) dt+\sigma_6(O-1)\, dM_6(t) \\
& +\left(1-\frac{1}{E_O}\right) dt+\sigma_7\left(E_O-1\right) dM_7(t) \\
& +\left(1-\frac{1}{I_{Oh}}\right) dt+\sigma_8\left(I_{Oh}-1\right) dM_8(t) \\
& +\left(1-\frac{1}{R}\right) dt+\sigma_9(R-1)\, dM_9(t) \\
& +\left(1-\frac{1}{D}\right) dt+\sigma_{10}(D-1)\, dM_{10}(t) \\
= & \alpha \Phi\left(S, E, I_a, I_s, I_h, O, E_O, I_{Oh}, R, D\right) dt+\sigma_1(S-1)\, dM_1(t) \\
& +\sigma_2(E-1)\, dM_2(t)+\sigma_3\left(I_a-1\right) dM_3(t)+\sigma_4\left(I_s-1\right) dM_4(t) \\
& +\sigma_5\left(I_h-1\right) dM_5(t)+\sigma_6(O-1)\, dM_6(t)+\sigma_7\left(E_O-1\right) dM_7(t) \\
& +\sigma_8\left(I_{Oh}-1\right) dM_8(t)+\sigma_9(R-1)\, dM_9(t)+\sigma_{10}(D-1)\, dM_{10}(t) .
\end{aligned}
$$

noting that $d\Phi : \mathbb{R}_{+}^{10} \rightarrow \mathbb{R}$ Replacing the right-side function of the system (3.1) into the above equation leads to

$$\begin{aligned}
& d\Phi\left(S, E, I_a, I_s, I_h, O, E_O, I_{Oh}, R, D\right) \\
= & \left(1-\frac{1}{S}\right)\left(\mu-\mu S(t)-\eta S(t)-\alpha_1 B_1\left(I_a(t)+I_s(t)\right) S(t)\right)+\frac{\sigma_1^2}{2} \\
& +\left(1-\frac{1}{E}\right)\left(\alpha_1 B_1\left(I_a(t)+I_s(t)\right) S(t)-\left(\beta_1+\beta_2+\beta_3+\mu\right) E(t)\right)+\frac{\sigma_2^2}{2} \\
& +\left(1-\frac{1}{I_a}\right)\left(\beta_1 E(t)-\mu I_a(t)-\gamma_1 I_a(t)\right)+\frac{\sigma_3^2}{2} \\
& +\left(1-\frac{1}{I_s}\right)\left(\beta_2 E(t)-\left(\mu+\gamma_2\right) I_s(t)\right)+\frac{\sigma_4^2}{2} \\
& +\left(1-\frac{1}{I_h}\right)\left(\beta_3 E(t)-\left(\mu+\gamma_3+\delta_1\right) I_h(t)\right)+\frac{\sigma_5^2}{2} \\
& +\left(1-\frac{1}{O}\right)\left(\eta S(t)-\mu O(t)-\alpha_2 B_2\left(I_a(t)+I_s(t)\right) O(t)\right)+\frac{\sigma_6^2}{2} \\
& +\left(1-\frac{1}{E_O}\right)\left(\alpha_2 B_2\left(I_a(t)+I_s(t)\right) O(t)-\left(\beta_4+\mu\right) E_O(t)\right)+\frac{\sigma_7^2}{2} \\
& +\left(1-\frac{1}{I_{Oh}}\right)\left(\beta_4 E_O(t)-\left(\mu+\gamma_4+\delta_2\right) I_{Oh}(t)\right)+\frac{\sigma_8^2}{2} \\
& +\left(1-\frac{1}{R}\right)\left(\gamma_1 I_a(t)+\gamma_2 I_s(t)+\gamma_3 I_h(t)+\gamma_4 I_{Oh}(t)-\mu R(t)\right)+\frac{\sigma_9^2}{2} \\
& +\left(1-\frac{1}{D}\right)\left(\delta_1 I_h(t)+\delta_2 I_{Oh}(t)\right)+\frac{\sigma_{10}^2}{2} \\
\leq & \; 11\mu+\eta+\alpha_1 B_1\left(I_a+I_s\right)+\alpha_1 B_1\left(I_a+I_s\right) S+\beta_1+\beta_2+\beta_3+\beta_1 E \\
& +\gamma_1+\beta_2 E+\gamma_2+\beta_3 E+\gamma_3+\delta_1+\eta S+\alpha_2 B_2\left(I_a+I_s\right)+\gamma_4 I_{Oh} \\
& +\alpha_2 B_2\left(I_a+I_s\right) O+\beta_4+\beta_4 E_O+\gamma_4+\delta_2+\gamma_1 I_a+\gamma_2 I_s+\gamma_3 I_h+\delta_1 I_h \\
& +\delta_2 I_{Oh}+\frac{\sigma_1^2+\sigma_2^2+\sigma_3^2+\sigma_4^2+\sigma_5^2+\sigma_6^2+\sigma_7^2+\sigma_8^2+\sigma_9^2+\sigma_{10}^2}{2} \\
= & \; K,
\end{aligned}$$

where K is positive and not dependent on time and $\widetilde{T}(t)$. Thus we have the following

$$\begin{aligned}
d\Phi\left(S, E, I_a, I_s, I_h, O, E_O, I_{Oh}, R, D\right) = & \; K dt+\sigma_1(S-1) dM_1(t)+\sigma_2(E-1) dM_2(t)+ \\
& +\sigma_3\left(I_a-1\right) dM_3(t)+\sigma_4\left(I_s-1\right) dM_4(t) \\
& +\sigma_5\left(I_h-1\right) dM_5(t)+\sigma_6(O-1) dM_6(t) \\
& +\sigma_7\left(E_O-1\right) dM_7(t)+\sigma_8\left(I_{Oh}-1\right) dM_8(t) \\
& +\sigma_9(R-1) dM_9(t)+\sigma_{10}(D-1) dM_{10}(t) .
\end{aligned} \tag{3.33}$$

Applying the integral to the equation above and then taking the expected value E on both sides, we obtain

$$E\left[\Phi\left(\widetilde{T}\left(\tau_\gamma \wedge \psi\right)\right)\right] \leq E\left[\Phi\left(\widetilde{T}(0)\right)\right]+K\psi. \tag{3.34}$$

Setting $F_\gamma=\tau_\gamma \leq \psi$ for $\gamma \geq \gamma_0$ and then $P\left(F_\gamma\right) \geq \varepsilon$.

For each $w \in F_\gamma$ there exist at least one of

$$S\left(\tau_\gamma \wedge \psi\right), E\left(\tau_\gamma \wedge \psi\right), I_a\left(\tau_\gamma \wedge \psi\right), I_s\left(\tau_\gamma \wedge \psi\right), I_h\left(\tau_\gamma \wedge \psi\right), O\left(\tau_\gamma \wedge \psi\right),$$

$$E_O\left(\tau_\gamma \wedge \psi\right), I_{Oh}\left(\tau_\gamma \wedge \psi\right), R\left(\tau_\gamma \wedge \psi\right), D\left(\tau_\gamma \wedge \psi\right),$$

equaling γ or $\frac{1}{\gamma}$. Then, we obtain

$$\Phi\left(\widetilde{T}\left(\tau_\gamma \wedge \psi\right)\right) \geq \left(\gamma - 1 - \ln\gamma\right) \wedge \left(\frac{1}{\gamma} - 1 - \ln\gamma\right). \tag{3.35}$$

In view of (3.34), one obtains

$$\begin{aligned} E\Phi\left(\widetilde{T}\left(0\right)\right) + K\psi &\geq E\left(1_{F_w}\Phi\left(\widetilde{T}\left(\tau_\gamma \wedge \psi\right)\right)\right) \\ &\geq \varepsilon\left\{\left(\gamma - 1 - \ln\gamma\right) \wedge \left(\frac{1}{\gamma} - 1 - \ln\gamma\right)\right\}, \end{aligned} \tag{3.36}$$

where 1_{F_w} stands for the indicator function of F_γ. As $\gamma \to \infty$, we obtain

$$\infty > E\Phi\left(\widetilde{T}\left(0\right)\right) + K\psi = \infty, \tag{3.37}$$

which is a contradiction. Thus, we conclude that $\tau_\infty = \infty$ with probability one, which completes the proof. □

In the following subsection, we will focus on specifying the conditions required for eliminating the spread of Covid-19 from the community with probability one [14, 4, 12, 3, 3, 16, 2].

3.3. Extinction of Covid-19:

To perform our aim, we want to define the following notation

$$\langle f\left(t\right)\rangle = \frac{1}{t}\int_0^t f\left(\tau\right)d\tau. \tag{3.38}$$

and also we present the following threshold parameters for our upcoming requirements:

$$\begin{aligned} R_0^{I_a} &= \frac{\beta_1}{\left(\mu + \gamma_1 + \frac{\sigma_3^2}{2}\right)}, R_0^{I_s} = \frac{\beta_2}{\left(\mu + \gamma_2 + \frac{\sigma_4^2}{2}\right)}, \\ R_0^{I_h} &= \frac{\beta_3}{\left(\mu + \gamma_3 + \frac{\sigma_4^2}{2}\right)}, R_0^{I_{Oh}} = \frac{\beta_4}{\left(\mu + \gamma_4 + \frac{\sigma_4^2}{2}\right)}. \end{aligned}$$

Definition 3.3. The spread of Covid-19 is said to become extinct with probability one if

$$\begin{aligned} \lim_{t\to\infty} I_a\left(t\right) &= 0, \text{almost surely (a.s.)} \\ \lim_{t\to\infty} I_s\left(t\right) &= 0, \text{ almost surely (a.s.)} \\ \lim_{t\to\infty} I_h\left(t\right) &= 0, \text{ almost surely (a.s.)} \\ \lim_{t\to\infty} I_{Oh}\left(t\right) &= 0, \text{ almost surely (a.s.).} \end{aligned} \tag{3.39}$$

Theorem 3.4. *Assume that $\widetilde{T}(t)$ is the solution of the Covid-19 model with initial data $\widetilde{T}(0) \in \mathbb{R}_+^{10}$. if $R_0^{I_a}, R_0^{I_s}, R_0^{I_h}, R_0^{I_{Oh}} < 1$, then we have*

$$\begin{aligned} \lim_{t\to\infty}\sup\frac{\ln I_a\left(t\right)}{t} &< 0, \quad (3.40)\\ \lim_{t\to\infty}\sup\frac{\ln I_s\left(t\right)}{t} &< 0, \\ \lim_{t\to\infty}\sup\frac{\ln I_h\left(t\right)}{t} &< 0, \\ \lim_{t\to\infty}\sup\frac{\ln I_{Oh}\left(t\right)}{t} &< 0, \quad (3.41) \end{aligned}$$

almost surely.

In other words, we can conclude that $I_a(t), I_s(t), I_h(t), I_{Oh}(t)$ approaches zero exponentially almost surely. This means that the Covid-19 will eventually be eliminated from the community with a probability of one.

Proof. We implement the integration on the infected classes to obtain

$$\begin{aligned}
\frac{I_a(t)-I_a(0)}{t} &= (\beta_1\langle E\rangle-(\mu+\gamma_1)\langle I_a\rangle)+\sigma_3\int I_a(\sigma)\,dM_3(\sigma),\\
\frac{I_s(t)-I_s(0)}{t} &= (\beta_2\langle E\rangle-(\mu+\gamma_2)\langle I_s\rangle)+\sigma_4\int I_s(\sigma)\,dM_4(\sigma),\\
\frac{I_h(t)-I_h(0)}{t} &= (\beta_3\langle E\rangle-(\mu+\gamma_3+\delta_1)\langle I_h\rangle)+\sigma_5\int I_h(\sigma)\,dM_5(\sigma),\\
\frac{I_{Oh}(t)-I_{Oh}(0)}{t} &= (\beta_4\langle E\rangle-(\mu+\gamma_4+\delta_2)\langle I_{Oh}\rangle)+\sigma_8\int I_{Oh}(\sigma)\,dM_8(\sigma).
\end{aligned}$$

To prove the theorem, we will apply Ito methodology on $\ln I_a(t), \ln I_s(t), \ln I_h(t), \ln I_{Oh}(t)$ and divide by t to obtain

$$\begin{aligned}
d\ln I_a(t) &= [\beta_1 E-(\mu+\gamma_1)I_a]\frac{1}{I_a}dt-\frac{\sigma_3^2}{2}dt+\sigma_3 dM_3(t) \\
&= \left[\beta_1\frac{E}{I_a}-\left(\mu+\gamma_1+\frac{\sigma_3^2}{2}\right)\right]dt+\sigma_3 dM_3(t),
\end{aligned} \tag{3.42}$$

$$\begin{aligned}
d\ln I_s(t) &= [\beta_2 E-(\mu+\gamma_2)I_s]\frac{1}{I_s}dt-\frac{\sigma_4^2}{2}dt+\sigma_4 dM_4(t)\\
&= \left[\beta_2\frac{E}{I_s}-\left(\mu+\gamma_2+\frac{\sigma_4^2}{2}\right)\right]dt+\sigma_4 dM_4(t),
\end{aligned}$$

$$\begin{aligned}
d\ln I_h(t) &= [\beta_3 E-(\mu+\gamma_3+\delta_1)I_h]\frac{1}{I_h}dt-\frac{\sigma_5^2}{2}dt+\sigma_5 dM_5(t)\\
&= \left[\beta_3\frac{E}{I_h}-\left(\mu+\gamma_3+\delta_1+\frac{\sigma_5^2}{2}\right)\right]dt+\sigma_5 dM_5(t)
\end{aligned}$$

$$\begin{aligned}
d\ln I_{Oh}(t) &= [\beta_4 E-(\mu+\gamma_4+\delta_2)I_{Oh}]\frac{1}{I_{Oh}}dt-\frac{\sigma_8^2}{2}dt+\sigma_8 dM_8(t)\\
&= \left[\beta_3\frac{E}{I_h}-\left(\mu+\gamma_4+\delta_2+\frac{\sigma_8^2}{2}\right)\right]dt+\sigma_8 dM_8(t).
\end{aligned}$$

We now apply Ito methodology on $\ln I_a(t), \ln I_s(t), \ln I_h(t), \ln I_{Oh}(t)$ and divide by t to obtain

$$\begin{aligned}
\frac{\ln I_a(t)-\ln I_a(0)}{t} &= \left[\beta_1\frac{E}{I_a}-\left(\mu+\gamma_1+\frac{\sigma_3^2}{2}\right)\right]+\frac{\sigma_3}{t}\int_0^t dM_3(\tau)\\
&\le \left(\mu+\gamma_1+\frac{\sigma_3^2}{2}\right)\left[\frac{\beta_1}{\left(\mu+\gamma_1+\frac{\sigma_3^2}{2}\right)}-1\right]+\frac{\sigma_3}{t}\int_0^t dM_3(\tau)\\
&\le \left(\mu+\gamma_1+\frac{\sigma_3^2}{2}\right)\left[R_0^{I_a}-1\right]<0.
\end{aligned} \tag{3.43}$$

$$\begin{aligned}\frac{\ln I_s(t)-\ln I_s(0)}{t} &= \left[\beta_2\frac{E}{I_s}-\left(\mu+\gamma_2+\frac{\sigma_4^2}{2}\right)\right]+\frac{\sigma_4}{t}\int_0^t dM_4(\tau)\\ &\le \left(\mu+\gamma_2+\frac{\sigma_4^2}{2}\right)\left[\frac{\beta_2}{\left(\mu+\gamma_2+\frac{\sigma_4^2}{2}\right)}-1\right]+\frac{\sigma_4}{t}\int_0^t dM_4(\tau)\\ &\le \left(\mu+\gamma_2+\frac{\sigma_4^2}{2}\right)\left[R_0^{I_s}-1\right]<0.\end{aligned}$$

$$\begin{aligned}\frac{\ln I_h(t)-\ln I_h(0)}{t} &= \left[\beta_3\frac{E}{I_h}-\left(\mu+\gamma_3+\delta_1+\frac{\sigma_5^2}{2}\right)\right]+\frac{\sigma_5}{t}\int_0^t dM_5(\tau)\\ &\le \left(\mu+\gamma_3+\delta_1+\frac{\sigma_5^2}{2}\right)\left[\frac{\beta_3}{\left(\mu+\gamma_3+\delta_1+\frac{\sigma_5^2}{2}\right)}-1\right]+\frac{\sigma_5}{t}\int_0^t dM_5(\tau)\\ &\le \left(\mu+\gamma_3+\delta_1+\frac{\sigma_5^2}{2}\right)\left[R_0^{I_h}-1\right]<0.\end{aligned}$$

$$\begin{aligned}\frac{\ln I_{Oh}(t)-\ln I_{Oh}(0)}{t} &= \left[\beta_4\frac{E_O}{I_s}-\left(\mu+\gamma_4+\delta_2+\frac{\sigma_8^2}{2}\right)\right]+\frac{\sigma_8}{t}\int_0^t dM_8(\tau)\\ &\le \left(\mu+\gamma_4+\delta_2+\frac{\sigma_8^2}{2}\right)\left[\frac{\beta_4}{\left(\mu+\gamma_4+\delta_2+\frac{\sigma_8^2}{2}\right)}-1\right]+\frac{\sigma_8}{t}\int_0^t dM_8(\tau)\\ &\le \left(\mu+\gamma_4+\delta_2+\frac{\sigma_8^2}{2}\right)\left[R_0^{I_{Oh}}-1\right]<0.\end{aligned}$$

Applying the superior limit, we obtain

$$\begin{aligned}\lim_{t\to\infty}\sup\frac{\ln I_a(t)}{t} &\le \left(\mu+\gamma_1+\frac{\sigma_3^2}{2}\right)\left[R_0^{I_a}-1\right]<0,\\ \lim_{t\to\infty}\sup\frac{\ln I_s(t)}{t} &\le \left(\mu+\gamma_2+\frac{\sigma_4^2}{2}\right)\left[R_0^{I_s}-1\right]<0,\\ \lim_{t\to\infty}\sup\frac{\ln I_h(t)}{t} &\le \left(\mu+\gamma_3+\delta_1+\frac{\sigma_5^2}{2}\right)\left[R_0^{I_h}-1\right]<0,\\ \lim_{t\to\infty}\sup\frac{\ln I_{Oh}(t)}{t} &\le \left(\mu+\gamma_4+\delta_2+\frac{\sigma_8^2}{2}\right)\left[R_0^{I_{Oh}}-1\right]<0,\end{aligned} \tag{3.44}$$

where

$$\begin{aligned}
\limsup_{t\to\infty}\left[\frac{\ln I_a(0)}{t}+\frac{\sigma_3}{t}\int_0^t dM_3(\tau)\right] &= 0, \\
\limsup_{t\to\infty}\left[\frac{\ln I_s(0)}{t}+\frac{\sigma_4}{t}\int_0^t dM_4(\tau)\right] &= 0, \\
\limsup_{t\to\infty}\left[\frac{\ln I_h(0)}{t}+\frac{\sigma_5}{t}\int_0^t dM_5(\tau)\right] &= 0, \\
\limsup_{t\to\infty}\left[\frac{\ln I_{Oh}(0)}{t}+\frac{\sigma_8}{t}\int_0^t dM_8(\tau)\right] &= 0.
\end{aligned} \tag{3.45}$$

This yields

$$\begin{aligned}
\lim_{t\to\infty} I_a(t) &= 0, \text{ almost surely (a.s.)}, \\
\lim_{t\to\infty} I_s(t) &= 0, \text{ almost surely (a.s.)}, \\
\lim_{t\to\infty} I_h(t) &= 0, \text{ almost surely (a.s.)}, \\
\lim_{t\to\infty} I_{Oh}(t) &= 0, \text{ almost surely (a.s.)}.
\end{aligned} \tag{3.46}$$

which completes the proof.

It is worth saying that it is concluded that the spread Covid-19 will be eradicated from the community with probability one. □

4. Numerical simulation

In this subsection, the model is further extended. Thus, the stochastic model will be solved numerically using a numerical scheme presented Runge-Kutta-4.The numerical simulations for these two scenarios (deterministic and stochastic) are presented in the Figure 1-10.

Here , the parameter represents the rate of new cases of obesity, while the natural birth rate of the population is denoted by . The transmission of the virus occurs when healthy individuals come into contact with asymptomatic and symptomatic infected individuals, with transmission rates represented by and , respectively. The infectiousness probabilities of symptomatic individuals, both non-obese and obese, are denoted by α_i. The parameters β_i represent the incubation period for newly infected individuals. The recovery rates for asymptomatic, symptomatic, non-obese hospitalized, and obese hospitalized individuals are given by γ_1, γ_2, γ_3, γ_4 , and , respectively. The disease-induced death rates for hospitalized individuals, both non-obese and obese, are represented by δ_1 and δ_2, respectively.

4.1. Effects of Model Parameters on Results

It is noteworthy that the transmission rate, represented by α_1, is predominantly influenced by non-pharmaceutical interventions such as physical distancing, mask-wearing, quarantine, and isolation measures for infected or exposed individuals, along with restrictions on non-essential gatherings and travel. Additionally, medical care plays a pivotal role in impacting the recovery rates (denoted as γ_i) for COVID-19. This encompasses access to appropriate medical facilities, equipment, and well-trained healthcare professionals. Moreover, the availability of effective treatments and medications capable of managing COVID-19 symptoms can significantly influence the recovery rate by reducing both the duration and severity of the illness. The primary aim of the simulations was to assess how various model parameters predominantly influenced the transmission dynamics and spread of COVID-19.

Table 1: Baseline parameter values.

Description	Notation	Baseline value	Reference
Population size	N	100000	Assumed
The natality rate	μ	5.644e-4(per day)	Assumed
The prevalence of obesity	η	1.6e-4(per day)	Assumed
The infection probability of a non-obese individual	α_1	0 – 0.30	Assumed
The infection probability of an obese individual	α_2	0 – 0.25	Assumed
The meeting rate of the non-obese individuals.	B_1	0.001 – 0.75	[18]
The meeting rate of the obese individuals.	B_2	0.001 – 0.30	[18]
The rate of progression from the exposed to the infectious stage	$(\beta_i)_{i=1,4}$	0 -0.5	[18]
The recovery rates	$(\gamma_i)_{i=1,4}$	0.001 – 0.1	[18]
The Covid-19 mortality rate of non-obese infected individuals	δ_1	0.0055	[18]
The Covid-19 mortality rate of infected obese individuals	δ_2	0.0099	Assumed
The stochastic constant	$(\sigma_\iota)_{\iota=1,10}$	0.002-0.08	Assumed

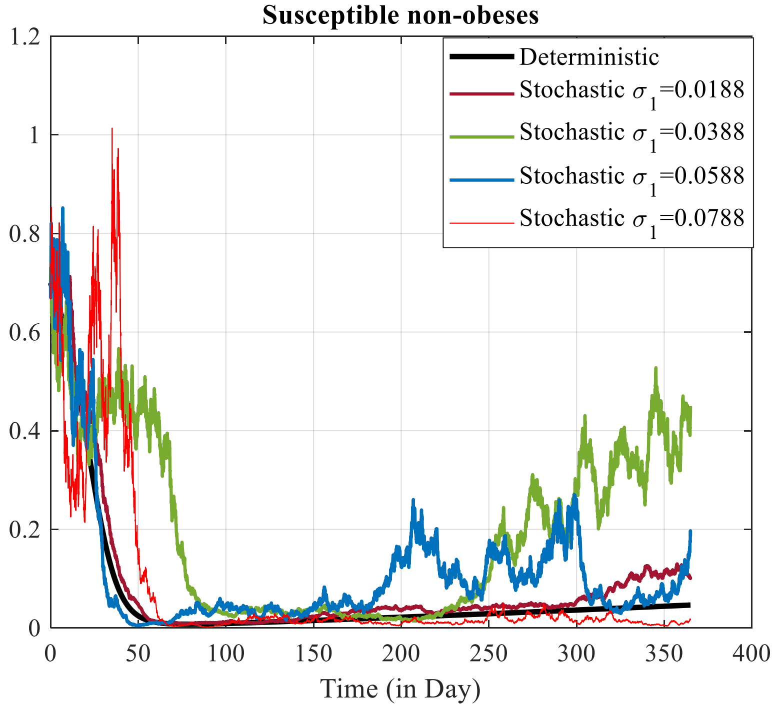

Figure 1: The graphical representation for the class $S(t)$.

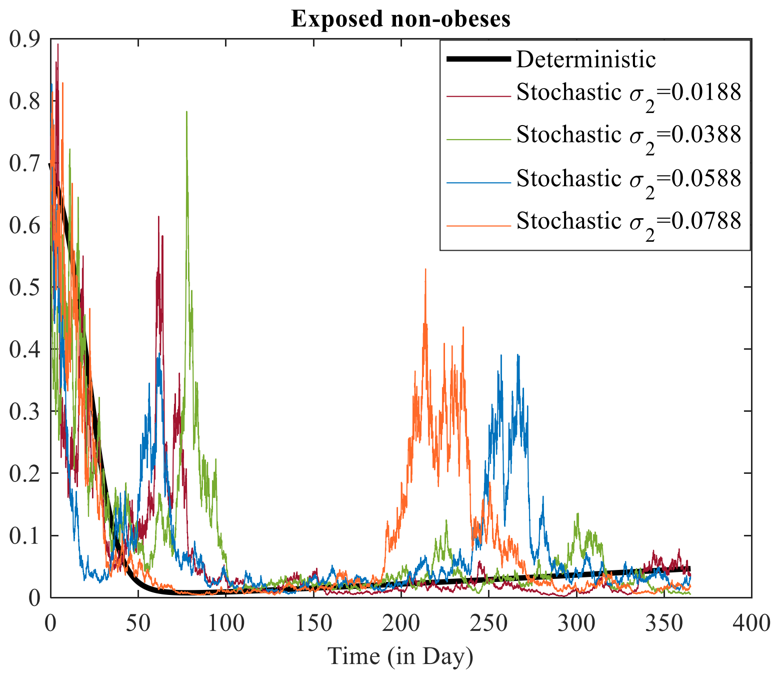

Figure 2: The graphical representation for the class $E(t)$.

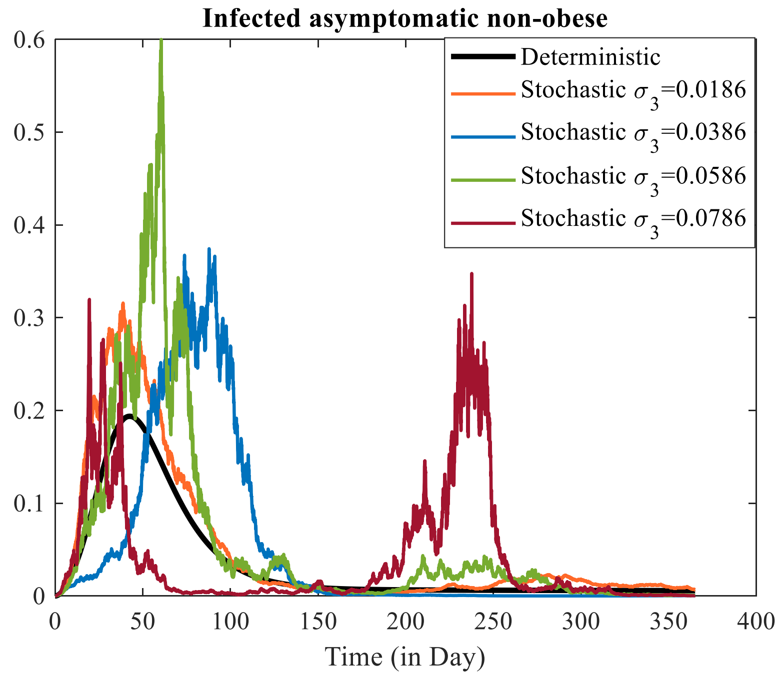

Figure 3: The graphical representation for the class $I_a\,(t)$.

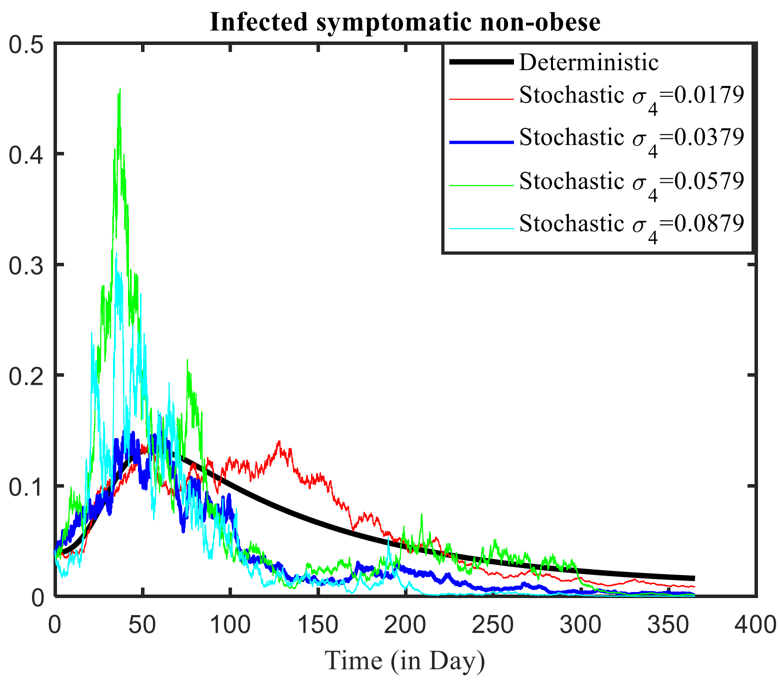

Figure 4: The graphical representation for the class $I_S\,(t)$.

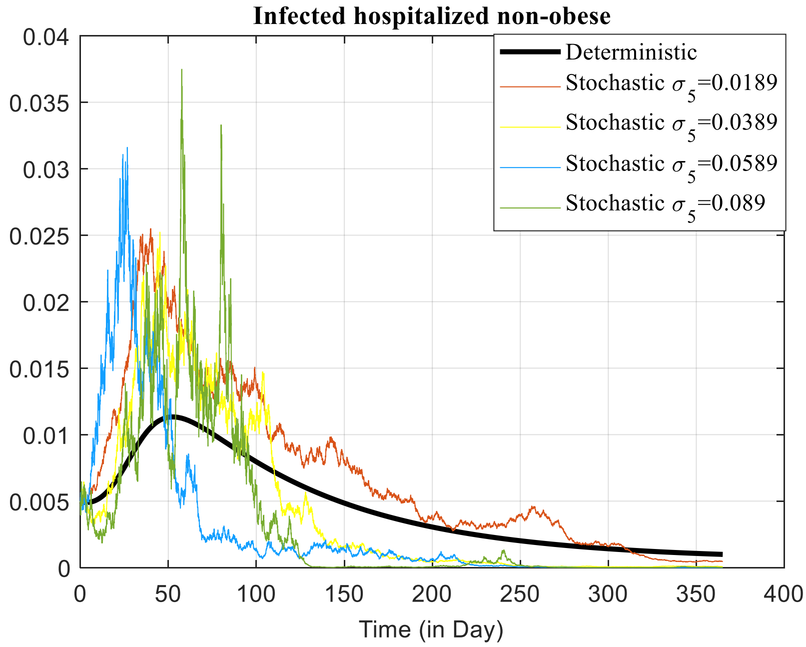

Figure 5: The graphical representation for the class $I_h(t)$.

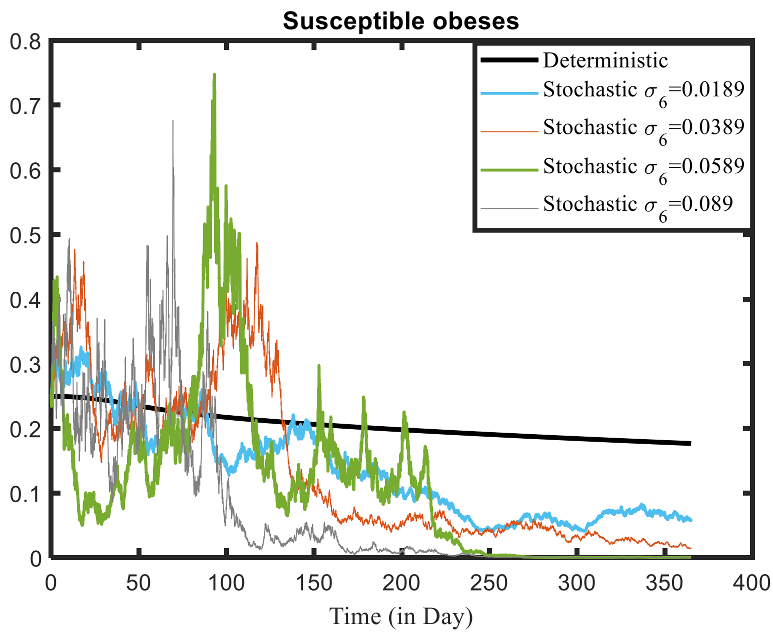

Figure 6: The graphical representation for the class $O(t)$.

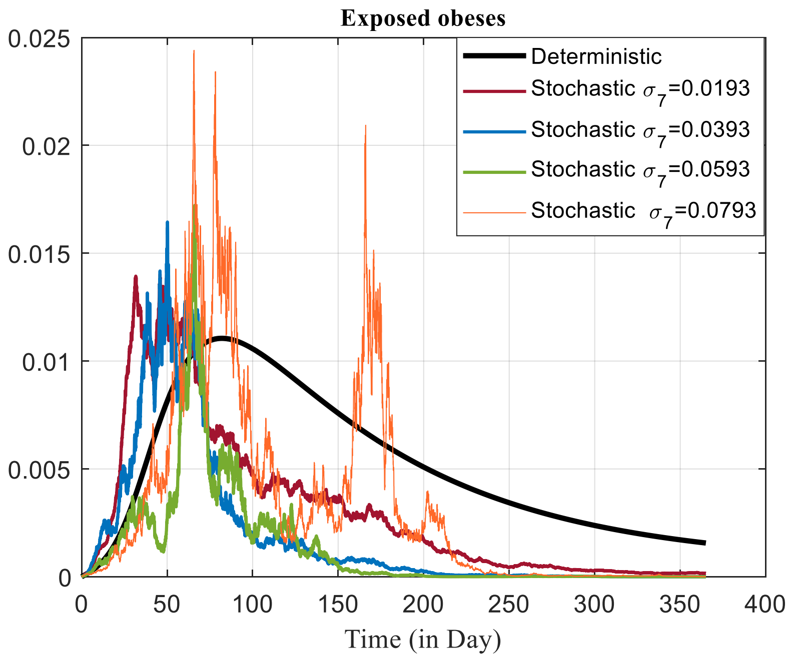

Figure 7: The graphical representation for the class $E_o\,(t)$.

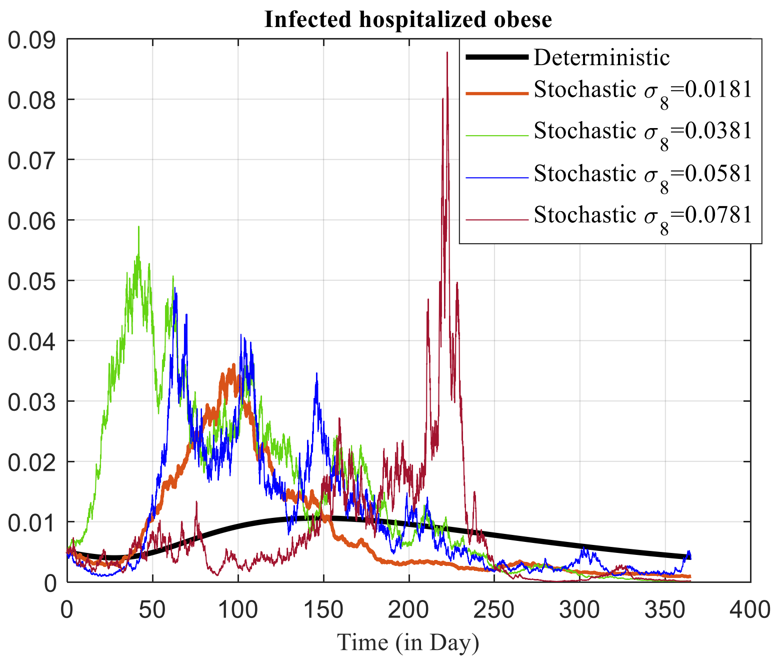

Figure 8: The graphical representation for the class $I_{oh}\,(t)$.

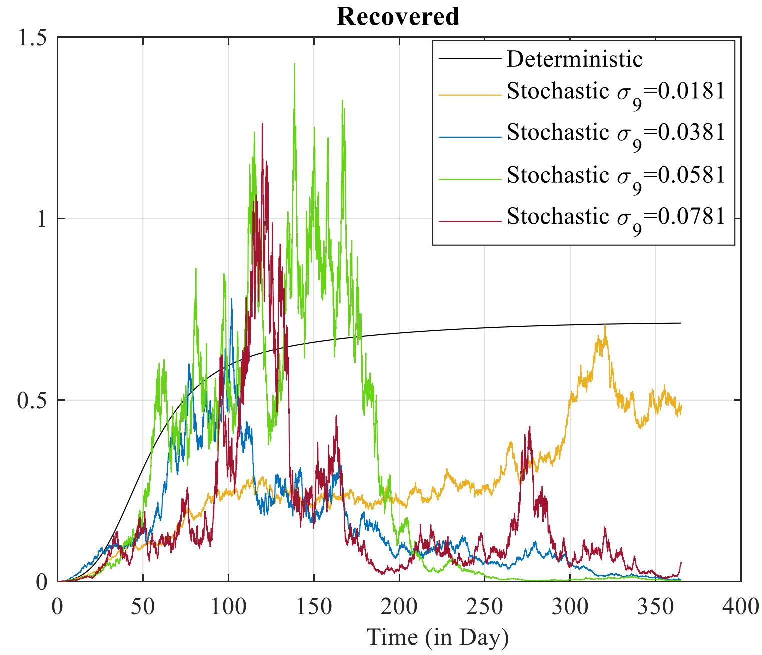

Figure 9: The graphical representation for the class $R(t)$.

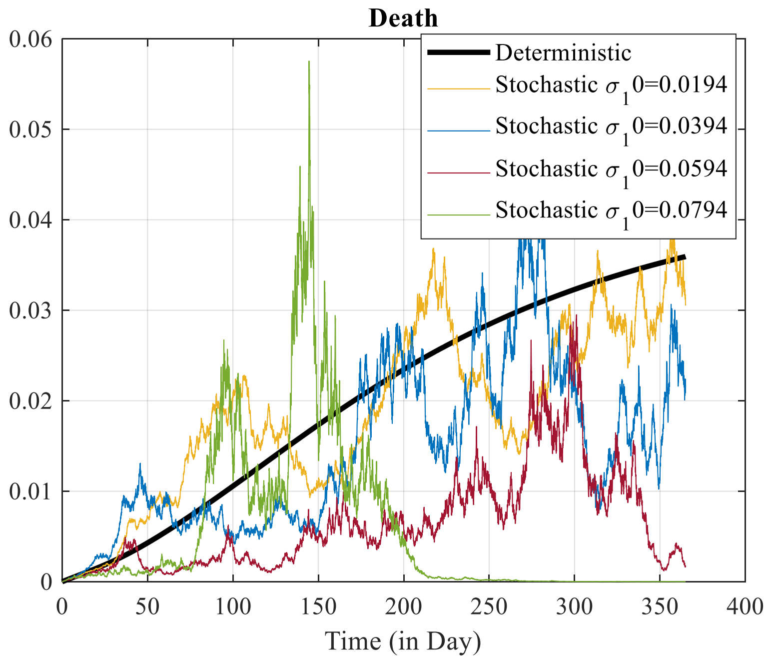

Figure 10: The graphical representation for the class $D(t)$.

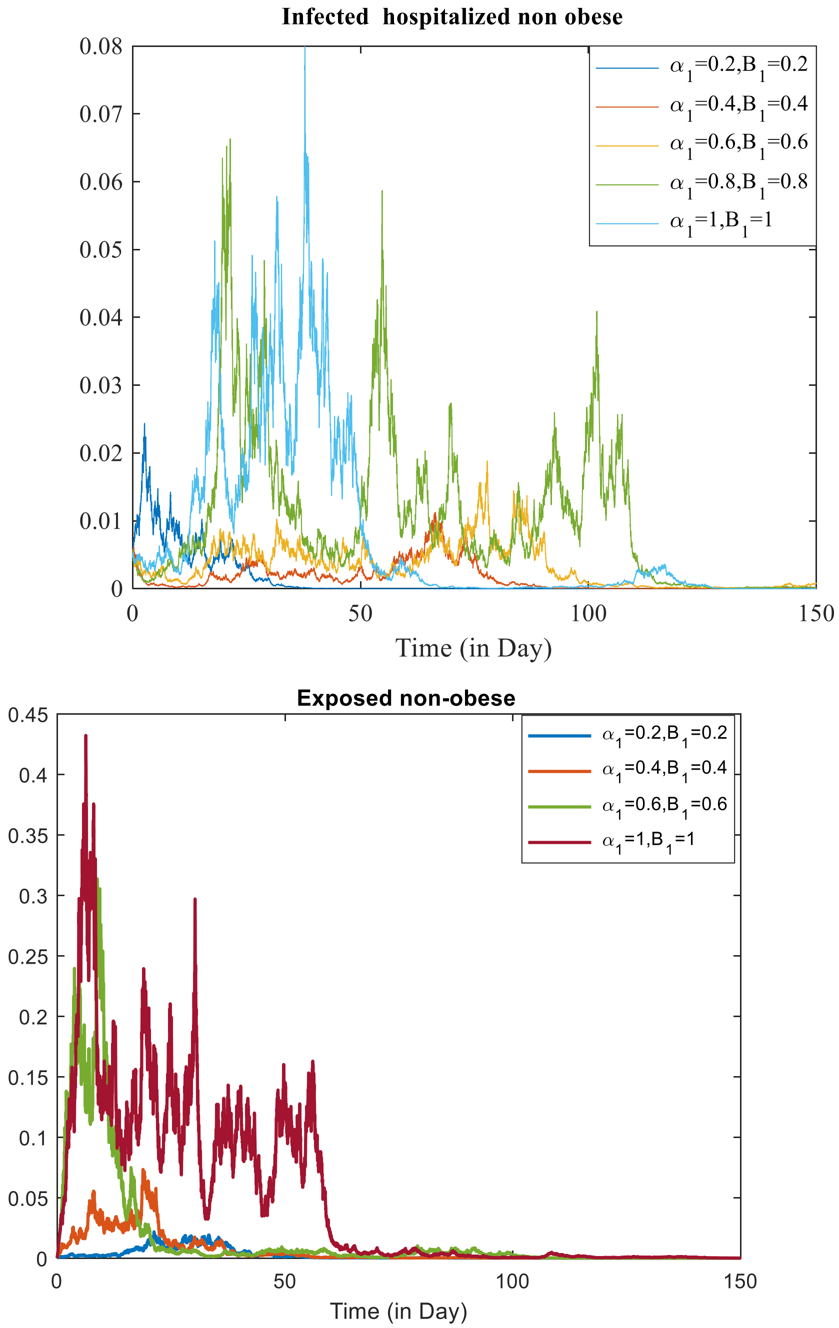

Figure 11: Effect of the $\alpha_1 B_1$ transmission rate on non-obese exposed and infected with COVID-19.

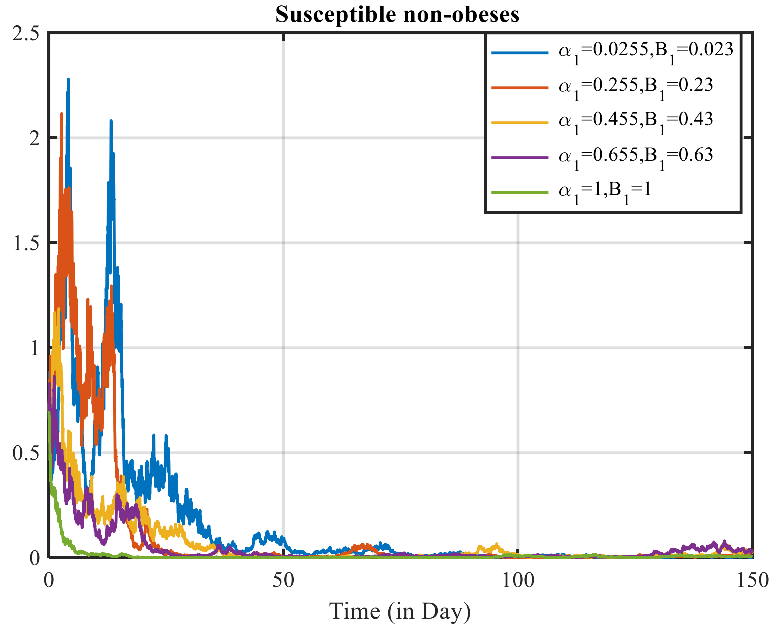

Figure 12: Effect of the $\alpha_1 B_1$ transmission rate on various states of COVID-19.

The numerical simulations depicted in figures (11,12,13) offer significant insights into the transmission dynamics and spread of COVID-19. These simulations underscore the crucial importance of managing the transmission rate, highlighting that recovery rates alone may not be sufficient to effectively reduce the spread of the virus. The results suggest that without stringent control measures, merely relying on the natural recovery process will not curb the disease's proliferation. Consequently, these findings can inform the development of comprehensive and effective strategies to control the transmission of COVID-19, thereby helping to mitigate its impact on public health and the economy.

5. Validation Analysis of the Model

A numerical simulation of the proposed model corroborates the findings reported by the Center for Systems Science and Engineering (CSSE) at Johns Hopkins University (January 21, 2022) [17]. This study focuses on data pertaining to hospitalizations and deaths due to COVID-19 from January 1 to March 1, 2021.

According to the Global Obesity Observatory [19], the United States has the highest obesity rate, while other countries have a moderate obesity rate. For the simulation, we will use parameters that characterize the population, including contact rates $B_{1,2}$, the healing rates γ_{1-4} and $\delta_{1,2}$ death rates.

Dynamics of hospitalized and death cases(the USA), with $B_1 = 0.32$, $B_2 = 0.11$, $\gamma_1 = 0.058$, $\gamma_2 = 0.1$, $\gamma_3 = 0.09$, $\gamma_4 = 0.091$, $\delta_1 = 0.03605$ and $\delta_2 = 0.0099$. The data considered in this work are hospitalized and death cases between January 1 and March 1, 2021 [17].

The results presented in figure 14 confirm that our findings align with those observed by CSSE [17]. A commonly used approach for comparing theoretical model results with experimental data is hypothesis testing. Hypothesis tests can be employed to compare the means and variances of the theoretical model results and experimental data to determine whether the simulation model has satisfactory accuracy for its intended application. The required accuracy of a model is typically specified as the difference between the model variables (means or variances) and those of the experimental data [13].

The procedure can be divided into the following four steps:

1. **Establish hypotheses and determine the significance level,** $\alpha = 0.05$:

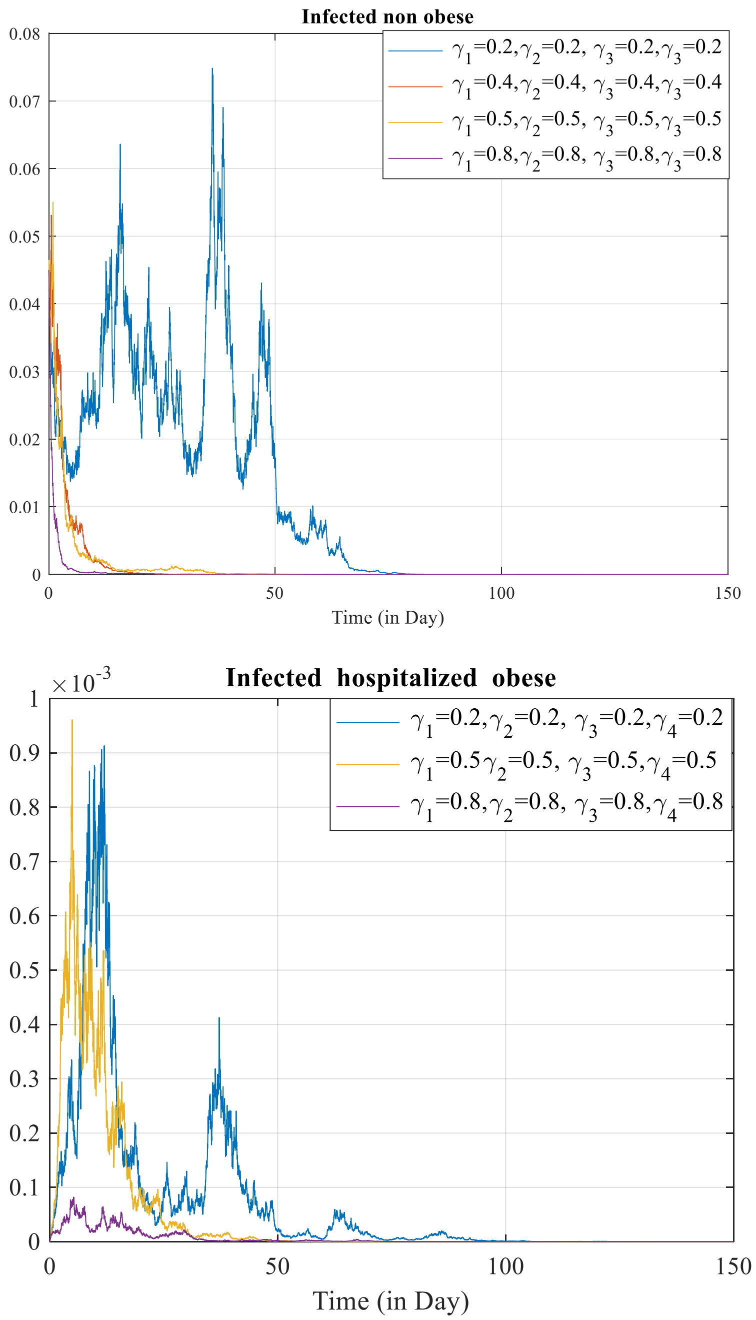

Figure 13: Effect of the γ_i on individuals infected with COVID-19.

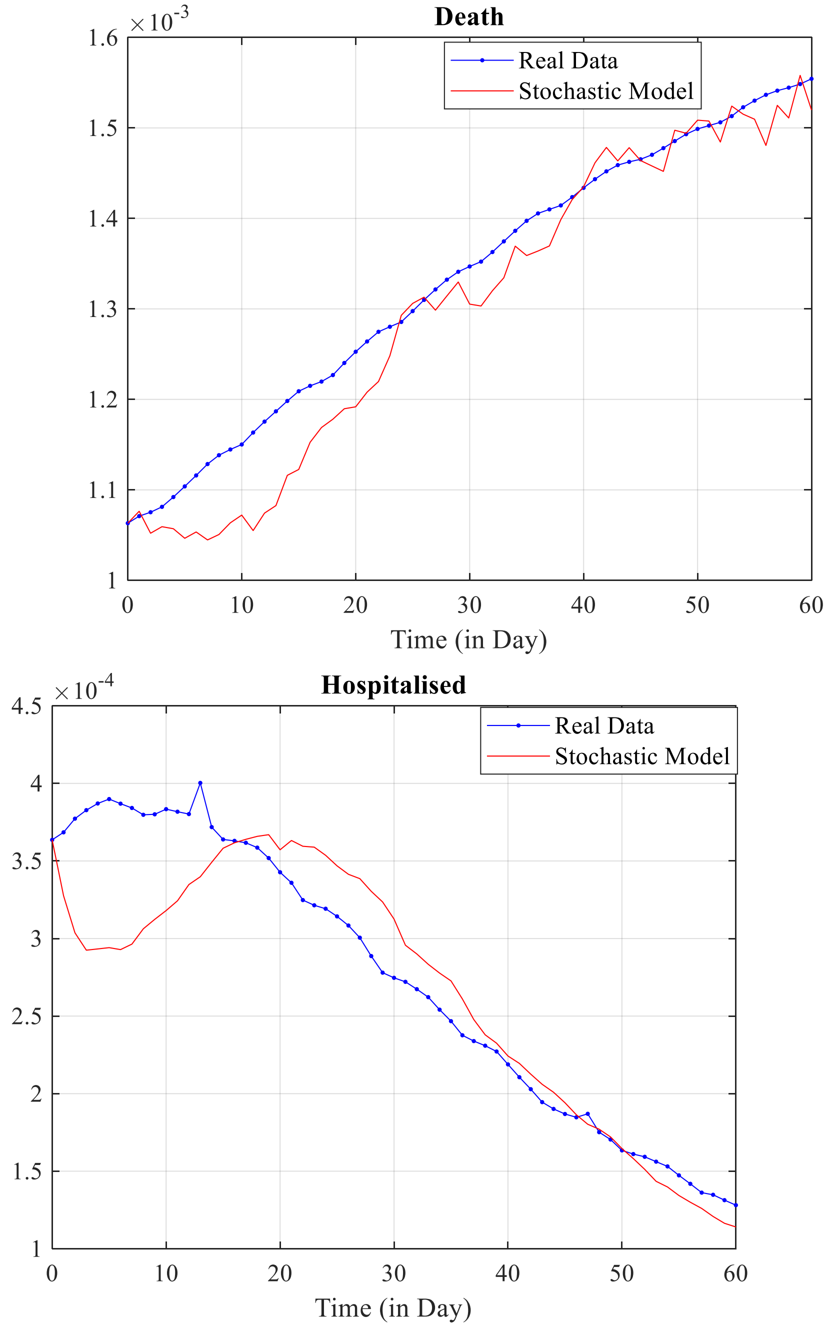

Figure 14: Dynamics of hospitalized and death cases(the USA), with $B_1 = 0.32$, $B_2 = 0.11$, $\gamma_1 = 0.058$, $\gamma_2 = 0.1$, $\gamma_3 = 0.09$, $\gamma_4 = 0.091$, $\delta_1 = 0.03605$ and $\delta_2 = 0.0099$. The data considered in this work are hospitalized and death cases between January 1 and March 1, 2021 [17].

H_0: There exists a strong agreement between the model and experimental data.
H_1: The model does not adequately fit the experimental data.

2. **Choose the appropriate test statistic:** To assess the adequacy of the theoretical model with experimental data in terms of both mean and variance, two tests are employed - the Student's T-test for mean and the Fisher-Snedecor's F-test for variance.
3. **Define the decision rule :** This involves calculating a p-value and comparing it with a predetermined threshold of 0.05[7] If

$$\begin{aligned} p-value\ calculated \quad &< \quad p-value(degrees\ of\ freedom\ of\ calculated \\ &\quad\quad data\ and\ observed\ data,\ 0.05\ threshold) \end{aligned}$$

then we fail to reject the null hypothesis of the test.

4. **Conclusion :** The ultimate conclusion is drawn by favorably interpreting the results of the T-test and F-test. Thus, at a 5% error risk, we accept the null hypothesis. Consequently, we can affirm a satisfactory alignment between the stochastic model (3.1) and the observed data sourced from the CSSE at Johns Hopkins University [17].

Table 2: Test for the USA results.

	F-value calculated	F-value at 0.05 threshold	Results	Conclusion
Hospitalized	0,4228	1.5288	We do not reject H_0.	There is no significant difference between the variance of hospitalized patients predicted by the stochastic model and the variance of observed hospitalized patients in the USA.
Death	0,000003203	1.5288	We do not reject H_0.	There is no significant difference between the variance of death patients predicted by the stochastic model and the variance of observed death patients in the USA.
	Student t-value calculated	**Student t-value at 0.05 threshold**	**Results**	**Conclusion**
Hospitalized	0,02584	0.4801	We do not reject H_0.	There is no significant difference between the mean of hospitalized patients predicted by the stochastic model and the mean of observed hospitalized patients in the USA.
Death	0,000003892	0.4801	We do not reject H_0.	There is no significant difference between the mean of death patients predicted by the stochastic model and the mean of observed death patients in the USA.

6. Conclusion

A groundbreaking mathematical model has been devised to understand the dynamics of coronavirus within the obese population, employing sophisticated stochastic differential equations. This innovative model not only accounts for the typical infected class but also introduces a specialized category for obese individuals who are infected. Moreover, a comprehensive examination of the epidemic's random behaviors, specifically within the context of COVID-19, has been undertaken through the lens of stochastic methodologies.

Our research demonstrates the efficacy of the stochastic model by establishing its global positive solution and delineating the circumstances wherein the disease might naturally dwindle within the population, leading to its extinction. To delve deeper into potential scenarios, extensive numerical simulations have been conducted. These simulations explore situations where interventions could inadvertently exacerbate or ameliorate the spread of infection, thus presenting strategies to shield the obese population from contagion and foster the restoration of healthy societal dynamics.

In conclusion, while deterministic models excel in predicting cumulative data trends, our findings underscore the superiority of stochastic models in accurately forecasting daily fluctuations in infection rates, offering invaluable insights for effective public health interventions. Additionally, it has been concluded that deterministic models are more proficient in forecasting cumulative data, whereas stochastic models excel in predicting daily data.

To finalize this work, we validated the proposed model against data observed in the USA, which has the highest obesity rate in the world. We found that the results of the proposed model were consistent with the observed data. ——————

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